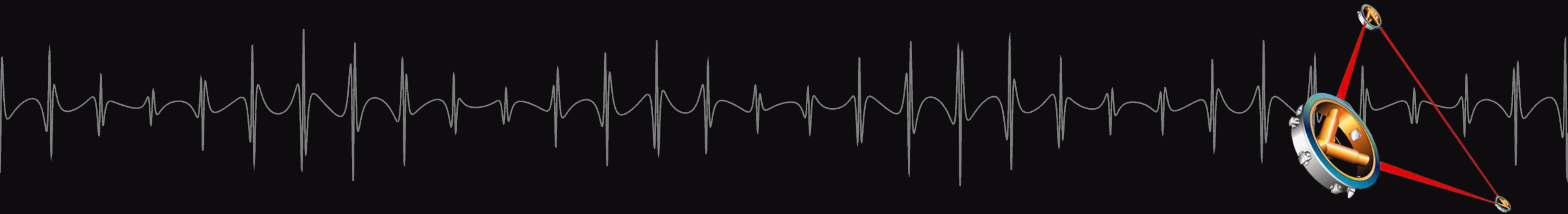

The road (and roadblocks) to EMRI search and inference

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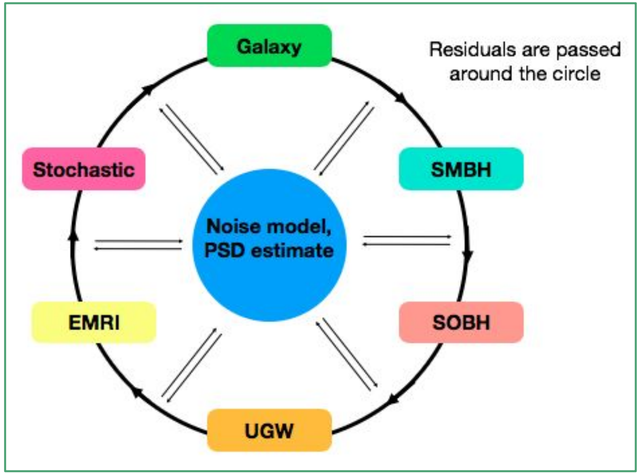
16 November 2020

EMRIs (Why are we on this road?)

- Extreme-mass-ratio inspirals are a key class of source for LISA
  - Capture of stellar-mass compact object (1-100 Solar) by massive BH (10^5-10^7 Solar)
  - Long-lived in LISA band (10^5 cycles); extreme precession; can be eccentric up to plunge
- The physics of EMRIs
  - Use BH perturbation theory with small mass ratio to calculate effective SF on Kerr orbits
  - Need SF up to 2nd-order dissipative; recent breakthrough at 2nd-order [Pound et al., 2020]
- The astrophysics of EMRIs
  - Uncertain event rates: 1-10^4 (per LISA) [Babak et al., 2017]
  - Brown-dwarf "problem" [Gourgoulhon et al., 2019; Amaro-Seoane, 2019]; other environmental effects
- Why bother? Environment may mess up modeling/analysis, or even existence
  - High-precision science: BH & galaxy astrophysics; tests of fundamental physics
  - Global fit: Even if LISA data contains just 1 EMRI signal, it will have to be accurately subtracted
  - Challenge: Everybody likes one

LISA data analysis (A map of the broader landscape)

- Waveforms & detector response
  - Long-lived signals: At least 3 years at 0.1 Hz (> 10^7 time samples)
  - TDI: Project strain onto evolving arms & cancel laser noise; difficult to do quickly & accurately
- The LISA global fit
  - Fully simultaneous vs Gibbs-style vs different rates?
  - Still many unknowns: Confusion among source types; convergence; noise estimation; candidate significance
- Gaps, glitches & non-stationary noise
  - 7-hour gaps every 2 weeks; optical-path & acceleration glitches; time-evolving noise PSD
  - Several recent studies [Robson & Cornish, 2019; Baghi et al., 2019; Edwards et al., 2020; Cornish, 2020]
  - TF methods are promising, but need development



N. Cornish

EMRI forward models (Choosing the right vehicle)

- Waveform can be decomposed into usual angular modes + frequency modes
  - Automatically handles precession & eccentricity, at the cost of dealing with many more modes
- Anatomy of a "bare-minimum" waveform for inference
  - Smooth* trajectory of generic Kerr geodesics with secular SF corrections accurate to 1PA order
  - Mode phasing with oscillatory SF corrections accurate to 1PA order (3 independent phases)
  - Mode amplitudes accurate to adiabatic order (10^5 independent amplitudes)

*Modulo resonances

$$G(t) \equiv (p(t), e(t), \iota(t))$$

$$\Phi_{mkn}(t) = \text{init.} + \int_{t_0}^{t} dt'\, \omega_{mkn}(G(t')) + \text{osc.} \qquad A_{lmkn}(t,\theta,\phi) = -2\frac{Z^{\infty}_{lmkn}(G(t))}{\omega^2_{mkn}(G(t))}\,{}_{-2}S_{lmkn}(\theta, G(t))e^{im\phi}$$

$$h_+ - ih_\times = \frac{1}{r}\sum_{lmkn} A_{lmkn}(t,\theta,\phi)e^{-i\Phi_{mkn}(t)}$$

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Difficult theory & computation (offline)

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Difficult computation (offline ~~& online~~)

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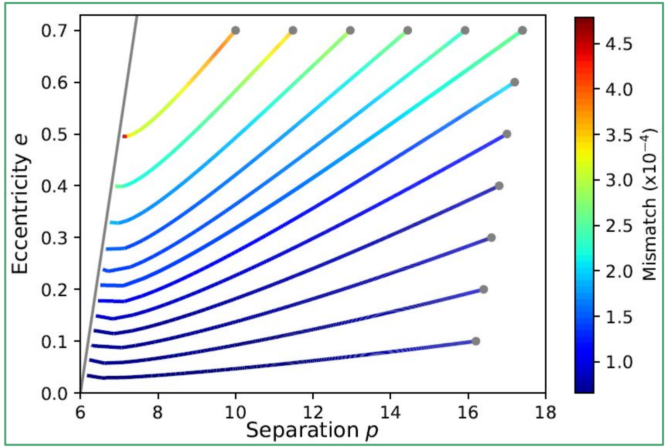
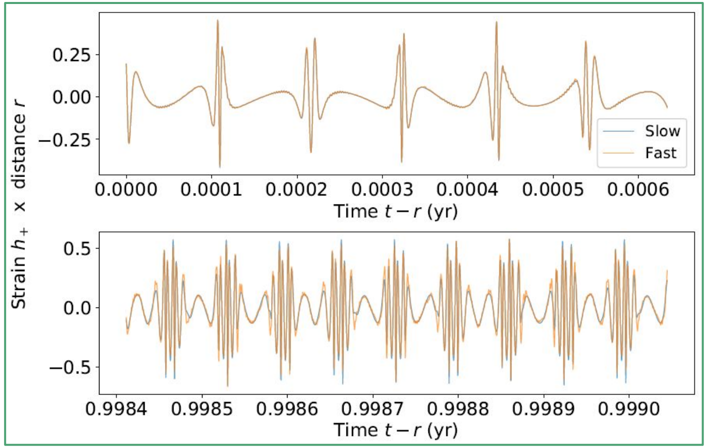
Difficult computation (online)

EMRI forward models (Choosing the right vehicle)

- Framework is implemented in FastEMRIWaveforms package (see Katz tutorial)
    - Accurate & efficient: Eccentric Schwarzschild; adiabatic [Chua et al., in rev.]
    - Efficient & extensive: Generic Kerr; semi-relativistic [Chua & Gair, 2015] (improved version)
    - “Accurate” & extensive: Generic Kerr; PN-adiabatic [Isoyama et al., in prep.] (not integrated yet)





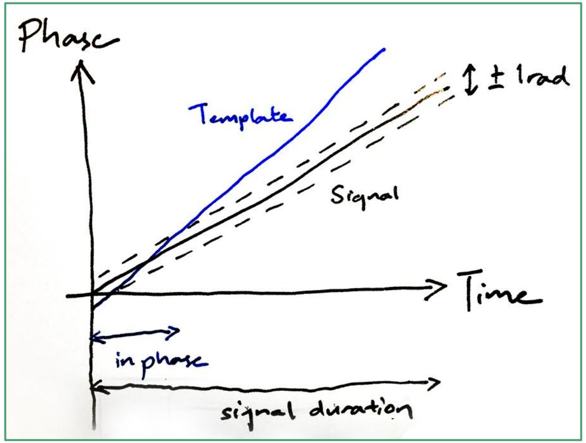
Chua et al., in rev.

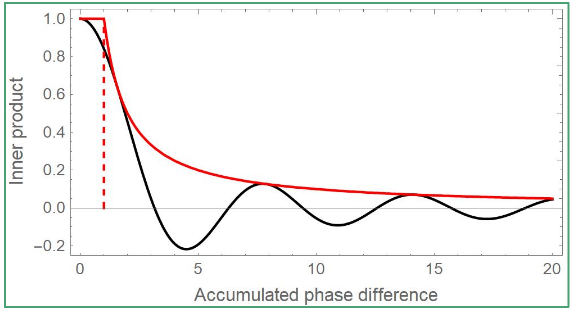
EMRI forward models (Choosing the right vehicle)

- Are there any alternative approaches to forward modeling? Yes, but...
- Time-domain solutions of field equations
  - Gold-standard in accuracy; very computationally expensive; relatively underdeveloped
  - Most practical model so far: GPU time-domain Teukolsky solver [Khanna & collaborators]
- Traditional ROM surrogates (of time-domain solutions)
  - Circular Schwarzschild IMRI: 1 parameter; $<$ 200 cycles; 22 modes [Rifat et al., 2020]
  - Unlikely to be data-analysis workhorse: Issues of accuracy & extensiveness
- Phenomenological models
  - Parametrize by mode amplitudes, frequencies & derivatives [Wang, Shang & Babak, 2012]
  - Main problem is mapping back to physical parameters, which still needs fast physical models
- What about environmental effects & modified GR?
  - Not a priority, but modular framework of FastEMRIWaveforms supports external development

EMRI search (Getting there)

- Space of LISA-observable EMRIs has gargantuan information volume
  - Hypothetical coverage with template bank requires 10^{40} templates [Gair et al., 2004]
- Hierarchical semi-coherent approach (motivated by LIGO CW searches)
  - Search with templates that are phase-maximized over number of time segments
  - Let’s use a phase-time plot to picture this for LIGO CWs or LISA GBs:





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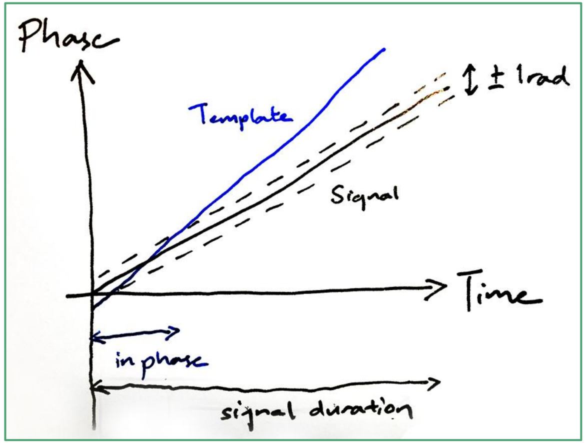
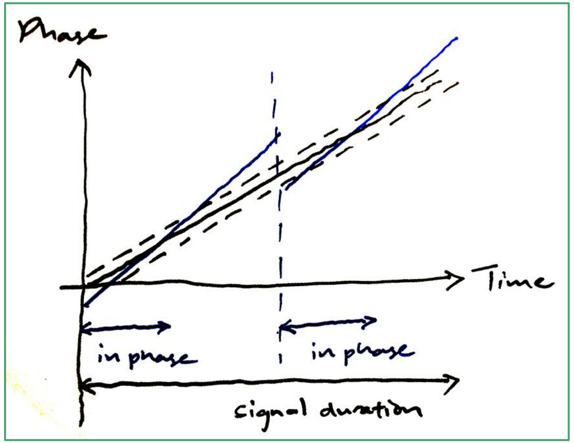
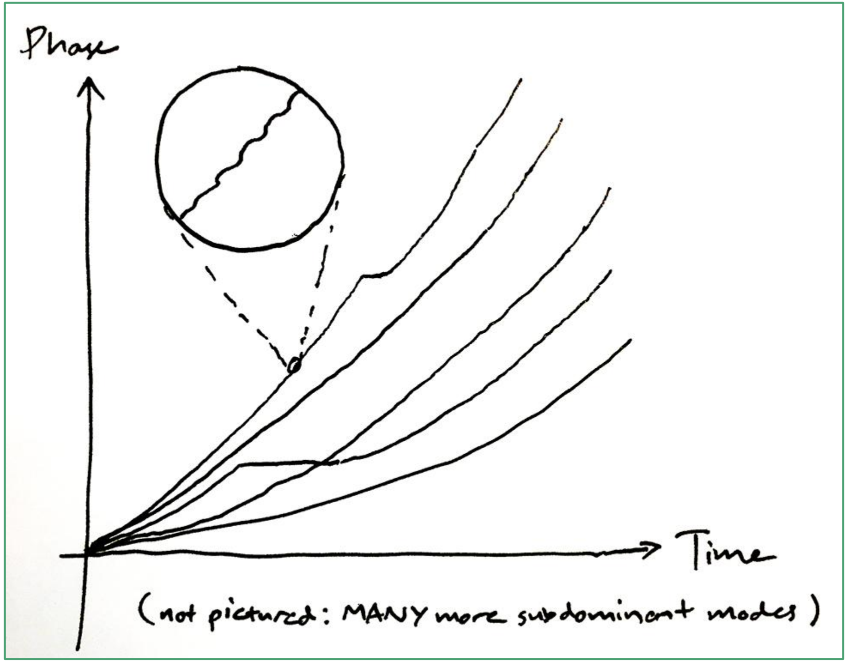




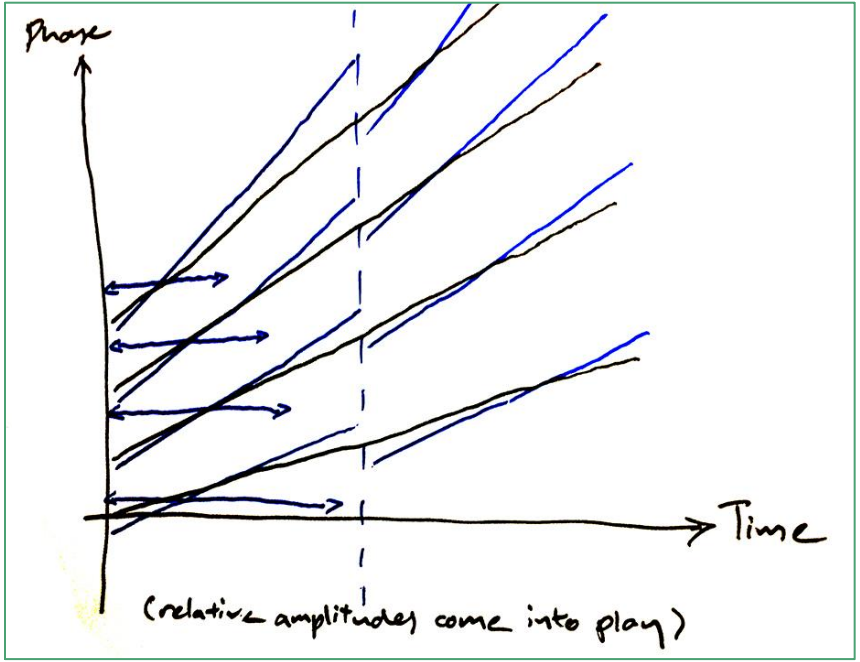
EMRI search (Getting there)

- What does an EMRI signal look like in the phase-time representation?



EMRI search (Getting there)

- But we can still play a similar game for EMRIs, to good approximation:

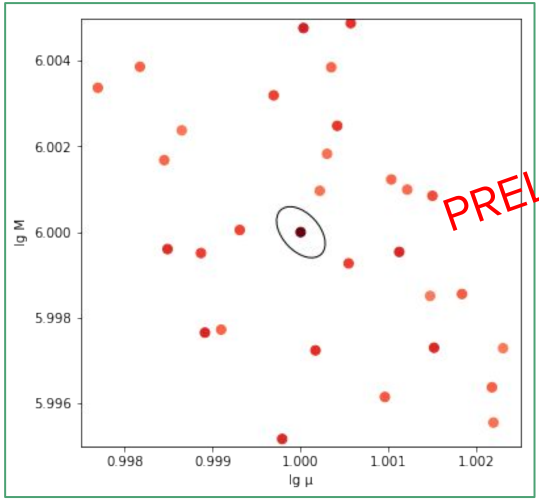


EMRI search (Getting there)

- Implicit assumption: Search model describes all possible signals
    - Holds for CWs & GBs: Signals are simple; observables are model parameters
- Does not hold for EMRIs: Plan is to use adiabatic waveforms for search
    - Effectively searching intersection between adiabatic & “true” (1PA) signal manifolds
    - Will sensitivity loss be acceptable? Localization could also be messed up
- Possible variation? Analyze segments independently; no secular information
    - Effectively searching larger manifold (parametrized by orbit at start of each segment)
    - Maybe can detect, but how to map back to initial orbit? Also increases information volume(!)
- What about minimally modeled or unmodeled searches?
    - Search with phenomenological models [Wang, Shang & Babak, 2012]
    - Semi-coherent phenomenological searches?
    - Search for excess power in TF data (spectrograms) [Gair & collaborators]

EMRI search (Getting there)

- Another roadblock: Is information volume really the problem per se?
- Parameter degeneracy in EMRI signal space [Chua & Cutler, in prep.]
  - Threshold-SNR (20) injection; 6 intrinsic parameters; posterior bounds × 10
  - 30 secondaries: Overlaps with injected signal range from 0.45 to 0.72



PRELIMINARY

EMRI search (Getting there)

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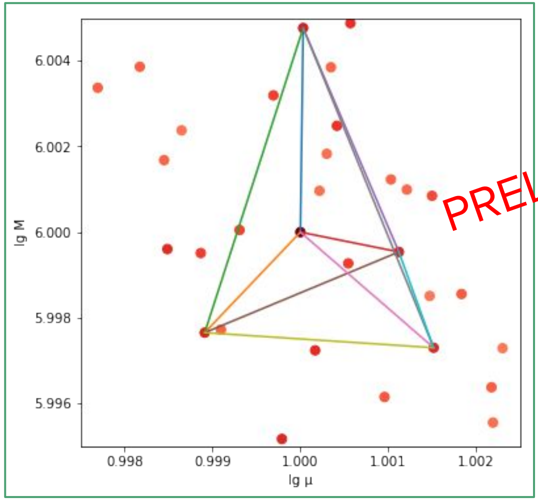
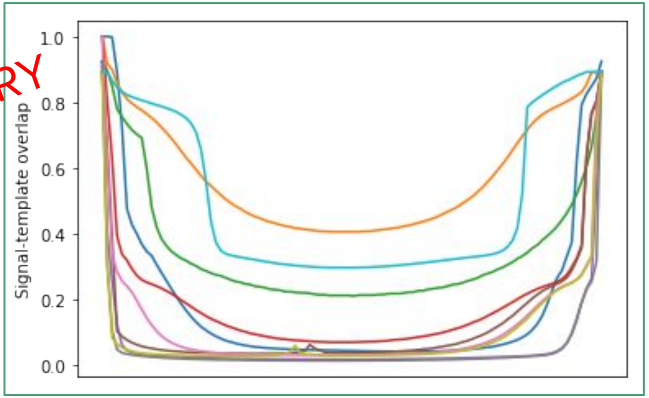
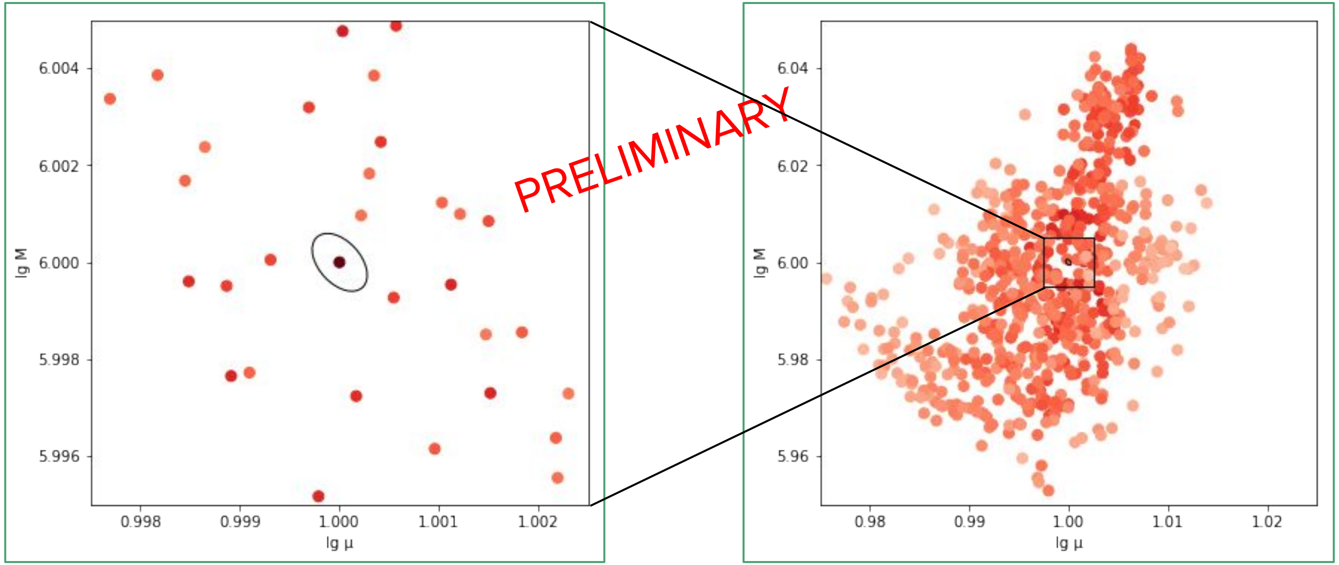


PRELIMINARY



EMRI search (Getting there)

- Secondary overlaps should fall off with distance from primary peak, right?
  - Same injection; posterior bounds × 100
  - 675 additional secondaries: Overlaps range from 0.23 to 0.76; evidence of undercounting

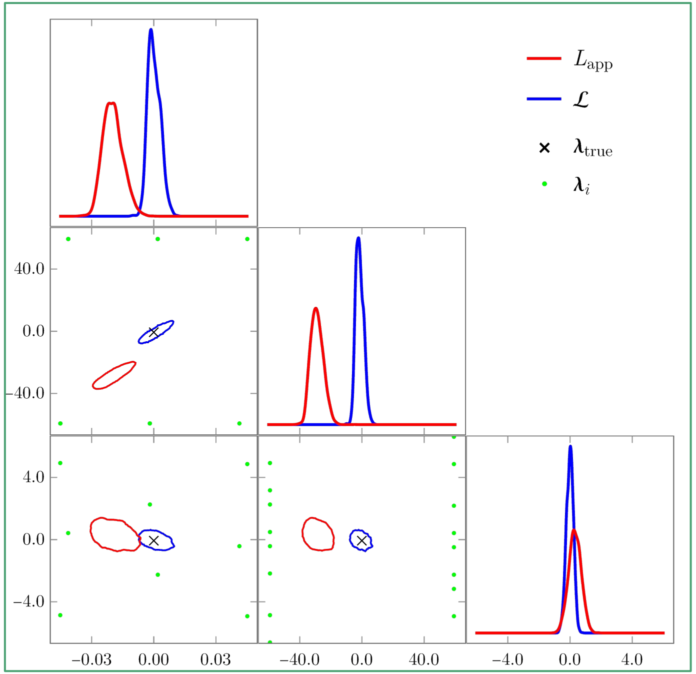


EMRI search (Getting there)

- Secondary + noise > primary?
  - Unlikely to be an issue: At threshold SNR, probability is < 1% if no secondary overlap > 0.78
- Sum of 2 secondaries from different signals > either primary?
  - Should not be an issue: Primaries are unlikely to coincide, so neither will secondaries(?)
  - More detailed analysis TBD
- Interaction with semi-coherent search?
  - Secondaries should congeal, but will they remain disconnected? Needs further investigation
- Main implication for now is sampling difficulty, which we already know
  - Degeneracy will not be addressed by “mode-hopping” MCMC proposals [Cornish, 2011]
  - Gradient-based sampling (e.g., HMC) will not help
  - Parallel tempering & nested sampling may work in principle, but will need high resolution

EMRI inference (Finding a parking spot)

- Inference is essentially end stage of search
- Fully coherent analysis is assumed
  - If forward modeling progresses as expected, standard approach should be within reach
  - Time- or TF-domain analysis needs development
- Degeneracy won't go away completely
  - Candidate regions must be sufficiently localized for standard samplers to start working
- Dealing with bias from model error
  - Estimate via Fisher [Cutler & Vallisneri, 2007]
  - Interpolate & marginalize over [Moore & Gair, 2014], but difficult for EMRIs [Chua et al., 2020]



Chua et al., 2020

Summary

- The road to EMRIs is paved with theoretical & computational difficulties
- This is in addition to the many distinctive challenges of LISA data analysis
- Several crucial considerations for EMRI forward modeling & search are underappreciated or still evolving; not just about scaling up standard methods
- EMRIs remain an exciting & open area of research!